APOGEE TECHNOLOGY, INC.

# *Filter Design Equations*

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## 1. Introduction

A generalized set of equations can be formulated for the design of first-order and second-order low pass and high pass filters. A specialized set of equations is devised for designing parametric biquad EQ filters. As with any other filter design procedure, the desired characteristics of the filter are to be made available. The following parameters governing the desired filter properties are typically given:

- $f_c$ – Cutoff/Center/Corner frequency
- Sampling rate ( $f_s$ ).
- Q factor ( $Q$ )
- $G =$ Boost/Cut dB gain value at $f_c$ (peaking and shelving filters)

The transfer function for a 1st order filter in digital z-domain can be written as:

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1}}{a_0 + a_1 \cdot z^{-1}} \qquad \text{(Eq. 1)}$$

And that for a biquad, the transfer function is defined as:

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{a_0 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2}} \qquad \text{(Eq. 2)}$$

Eq. 2 shows the basic transfer function, however the most commonly used transfer function used in the implementation of Direct Form I filter structure is shown below:

$$H(z) = \frac{(b_0/a_0) + (b_1/a_0) \cdot z^{-1} + (b_2/a_0) \cdot z^{-2}}{1 + (a_1/a_0) \cdot z^{-1} + (a_2/a_0) \cdot z^{-2}} \qquad \text{(Eq. 3)}$$

From this transfer function, the underlying difference equation is derived to be:

$$y[n] = \{(b_0/a_0) \cdot x[n]\} + \{(b_1/a_0) \cdot x[n-1]\} + \{(b_2/a_0) \cdot x[n-2]\} - \{(a_1/a_0) \cdot y[n-1]\} - \{(a_2/a_0) \cdot y[n-1]\}$$

(Eq. 4)

## 2. 1st order IIR Filter Design

First compute a few intermediate variables to use in the design:

- $\omega_c = 2 \cdot \pi \cdot f_c / f_s$
- $\mathrm{K} = \tan(\omega_c/2)$
- $\alpha = 1 + \mathrm{K} = 1 + \tan(\omega_c/2)$

The denominator coefficients are identical for both low-pass and high-pass filters designed for the same cut-off frequency and are computed as follows:

- $a_0 = 1$
- $a_1 = -[(1 - \mathrm{K})/\alpha]$

The numerator coefficients for a low-pass filter can be calculated as follows:

- $b_0 = b_1 = \mathrm{K}/\alpha$

The numerator coefficients for a high-pass filter can be calculated as follows:

- $b_0 = 1/\alpha$
- $b_1 = -1/\alpha$

## 3. 2nd order IIR Filter Design

Let us first compute a few intermediate variables:

- $A = \sqrt{10^{(G/20)}}$ (For peaking and shelving EQ filters only)
- $\omega_c = 2 \cdot \pi \cdot f_c / f_s$
- $\omega S = \sin(\omega_c)$
- $\omega C = \cos(\omega_c)$
- $\alpha = \omega S / (2 \cdot Q)$
- $\beta = \sqrt{A} / Q$ (For shelving EQ filters only)

The following sections now list the equations to be followed in order compute the coefficients for the desired filter type.

**Low Pass Filter**

- $b_0 = (1 - \omega C)/2$
- $b_1 = 1 - \omega C$
- $b_2 = (1 - \omega C)/2$
- $a_0 = 1 + \alpha$
- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - \alpha$

**High Pass Filter**

- $b_0 = (1 + \omega C)/2$
- $b_1 = -(1 + \omega C)$
- $b_2 = (1 + \omega C)/2$
- $a_0 = 1 + \alpha$
- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - \alpha$

**Band Pass Filter**

Peak gain = Q

- $b_0 = \omega S/2 = Q \cdot \alpha$
- $b_1 = 0$
- $b_0 = -\omega S/2 = -Q \cdot \alpha$
- $a_0 = 1 + \alpha$
- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - \alpha$

Constant 0 dB peak gain

- $b_0 = \alpha$
- $b_1 = 0$
- $b_2 = -\alpha$
- $a_0 = 1 + \alpha$
- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - \alpha$

**Notch Filter**

- $b_0 = 1$
- $b_1 = -2 \cdot \omega C$
- $b_2 = 1$
- $a_0 = 1 + \alpha$
- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - \alpha$

**All Pass Filter**

- $b_0 = 1 - \alpha$
- $b_1 = -2 \cdot \omega C$
- $b_2 = 1 + \alpha$
- $a_0 = 1 + \alpha$

- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - \alpha$

**Peaking EQ Filter**

- $b_0 = 1 + (\alpha \cdot A)$
- $b_1 = -2 \cdot \omega C$
- $b_2 = 1 - (\alpha \cdot A)$
- $a_0 = 1 + (\alpha / A)$
- $a_1 = -2 \cdot \omega C$
- $a_2 = 1 - (\alpha / A)$

**Low Shelving Filter**

- $b_0 = A \cdot \{(A+1) - [(A-1) \cdot \omega C] + (\beta \cdot \omega S)\}$
- $b_1 = 2 \cdot A \cdot \{(A-1) - [(A+1) \cdot \omega C]\}$
- $b_2 = A \cdot \{(A+1) - [(A-1) \cdot \omega C] - (\beta \cdot \omega S)\}$
- $a_0 = \{(A+1) + [(A-1) \cdot \omega C] + (\beta \cdot \omega S)\}$
- $a_1 = -2 \cdot \{(A-1) + [(A+1) \cdot \omega C]\}$
- $a_2 = \{(A+1) + [(A-1) \cdot \omega C] - (\beta \cdot \omega S)\}$

**High Shelving Filter**

- $b_0 = A \cdot \{(A+1) + [(A-1) \cdot \omega C] + (\beta \cdot \omega S)\}$
- $b_1 = -2 \cdot A \cdot \{(A-1) + [(A+1) \cdot \omega C]\}$
- $b_2 = A \cdot \{(A+1) + [(A-1) \cdot \omega C] - (\beta \cdot \omega S)\}$
- $a_0 = \{(A+1) - [(A-1) \cdot \omega C] + (\beta \cdot \omega S)\}$
- $a_1 = 2 \cdot \{(A-1) - [(A+1) \cdot \omega C]\}$
- $a_2 = \{(A+1) - [(A-1) \cdot \omega C] - (\beta \cdot \omega S)\}$

## 4. Filter Stability Conditions

For a second order filter, two conditions need to be satisfied to ensure filter stability. A filter is said to be stable in the z-domain if the roots/poles of the filter lie inside the unit circle. This definition of stability can be translated in terms of the filter coefficients to take the form:

- $|a_2| < 1$
- $|a_1| < 1 + a_2$

For a first-order filter, the stability condition that needs to be satisfied is that the pole of the filter lies within the unit circle. Again writing in terms of the coefficients just designed, the condition can be given as:

- $|a_1| < 1$

The first-order and second-order systems are stable *if and only if* the requisite conditions have been satisfied.

## 5. Steps to obtain DDX-4100(A) usable filter coefficients

The filter coefficients obtained using the design equations are not compatible with the format used by the DDX-4100(A) applications. The coefficients are in the form of signed fractional numbers, whereas the controller registers require the coefficients to be in a signed fixed-point integer format.

The DDX-4100 uses the following implementation of the biquad filter:

$$y[n] = x[n] + \{(b_0/a_0) - 1\} \cdot x[n] + 2 \cdot \{(b_1/a_0)/2\} \cdot x[n-1] + \{(b_2/a_0) \cdot x[n-2]\}$$
$$-2 \cdot \{(a_1/a_0)/2\} \cdot y[n-1] - \{(a_2/a_0) \cdot y[n-2]\}$$

(Eq. 5)

The filter implementation given above assumes that $a_0$ = 1. Therefore for a non-unity $a_0$ coefficient, in order to retain the frequency characteristics of the desired filter, it is required that the other five coefficients be scaled by this $a_0$ coefficient value, as shown below. The order and format in which the coefficients are to be provided is as follows:

- $b_2/a_0$
- $(b_0/a_0) - 1$
- $a_2/a_0$
- $(a_1/a_0)/2$
- $(b_1/a_0)/2$

The coefficients are constrained to be in a 20-bit signed fixed-point integer format. Hence the filter coefficients must be in the range [80000h, 7FFFFh], which is equivalent to the fractional range [-1.0, 1.0]. In order to meet the afore mentioned criteria, the filter coefficients computed using the design equations are first processed as per the relations given above. This set of newly processed coefficients are quantized to 20-bit fixed-point numbers and then converted into hexadecimal format. Now, the coefficients are ready to be loaded into the DDX-4100(A) design using the GUI.

## 6. Steps to Obtain DDX-8000 usable filter coefficients

The filter coefficients obtained using the design equations are not compatible with the format used by the DDX-8000 applications. The coefficients are in the form of signed fractional numbers, whereas the controller registers require the coefficients to be in a signed fixed-point integer format.

The DDX-8000 uses the following implementation of the biquad filter:

$$y[n] = 2\cdot\{(b_0/a_0)/2\}\cdot x[n] + 2\cdot\{(b_1/a_0)/2\}\cdot x[n-1] + \{(b_2/a_0)\cdot x[n-2]\}$$
$$+ 2\cdot\{(-a_1/a_0)/2\}\cdot y[n-1] + \{(-a_2/a_0)\cdot y[n-2]\}$$

(Eq. 6)

The filter implementation given above assumes that $a_0$ = 1. Therefore for a non-unity $a_0$ coefficient, in order to retain the frequency characteristics of the desired filter, it is required that the other five coefficients be scaled by this $a_0$ coefficient value, as shown below. The order and format in which the coefficients are to be provided is as follows:

- $b_2/a_0$
- $(b_0/a_0)/2$
- $-a_2/a_0$
- $-(a_1/a_0)/2$
- $(b_1/a_0)/2$

The coefficients are constrained to be in a 24-bit signed fixed-point integer format. Hence the filter coefficients must be in the range [800000h, 7FFFFFh], which is equivalent to the fractional range [-1.0, 1.0]. In order to meet the afore mentioned criteria, the filter coefficients computed using the design equations are first processed as per the relations given above. This set of newly processed coefficients are quantized to 24-bit fixed-point numbers and then converted into hexadecimal format. Now, the coefficients are ready to be loaded into the DDX-8000 design using the GUI.

## 7. Example

Say, a Low Pass Filter with the following specifications is to be designed.

- $f_s = 192000 Hz$
- $f_c = 150 Hz$
- $Q = 0.707$

By default, for high-pass and low-pass filters, the parameter $G$ is of no significance. Now, as per the above equations the following variables are calculated:

- $\omega_c = 2\cdot\pi\cdot f_c/f_s = 0.00490873852123$
- $\omega S = \sin(\omega_c) = 0.004908718808$
- $\omega C = \cos(\omega_c) = 0.99998795216726$
- $\alpha = \omega S/(2\cdot Q) = 0.00347151259406$

- $b_0 = (1 - \omega_c)/2 = 0.000006023916371556$
- $b_1 = 1 - \omega_c = 0.000012047832743112$
- $b_2 = (1 - \omega_c)/2 = 0.000006023916371556$
- $a_0 = 1 + \alpha = 1.00347151259406$
- $a_1 = -2 \cdot \omega C = \text{-}1.99997590433451$
- $a_2 = 1 - \alpha = 0.99652848740594$

Now, to convert these coefficients into a usable format for say, DDX-8000 controller. In reference to Eq.6 and based on the relations given in section 6, the coefficients calculated above must be processed as follows:

- $b_2/a_0 = 0.00000600307662$
- $(b_0/a_0)/2 = 0.00000300153831$
- $-a_2/a_0 = \text{-}0.99308099422756$
- $-(a_1/a_0)/2 = 0.99652849096055$
- $(b_1/a_0)/2 = 0.00000600307662$

Next, we convert these fractional numbers into quantized signed 24-bit fixed-point numbers and then to corresponding hexadecimal values to be used in the DDX-8000.

- $b_2/a_0 = 50 = 0x000032$
- $(b_0/a_0)/2 = 25 = 0x000019$
- $-a_2/a_0 = -8330568 = 0x80E2B8$
- $-(a_1/a_0)/2 = 8359486 = 0x7F8E3E$
- $(b_1/a_0)/2 = 50 = 0x000032$

## 8. References

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